University of California, Berkeley
Physics H7C Fall 1999 (*Strovink*)

PROBLEM SET 4

1.
A plane wave polarized in the $\hat{\mathbf{x}}$ direction is normally incident upon an ideal quarter-wave plate which has its slow axis oriented at $+45°$ with respect to the $\hat{\mathbf{x}}$ axis. Next it is reflected at normal incidence by a perfectly conducting mirror. Finally the wave passes back through the same quarter-wave plate. After completing this journey...
(a.)
What is the final wave's irradiance, relative to the incident wave?
(b.)
What is the final wave's state of polarization? (If linearly polarized, specify the polarization direction; if circularly polarized, state whether right- or left-handed.)
Both of your answers should be justified.

2.
A fiber-optic cable consists of a cylindrical core with refractive index n_1 surrounded by a sheath with refractive index n_2, where $1 < n_2 < n_1$. The ends of the cable are cut perpendicular to the cable axis and polished. A point source of light is placed on the cable axis a negligible distance away from one end. What fraction of the total light emitted by the point source is transmitted by the core to the (distant) end of the cable? (You may ignore losses due to reflection at the cable ends.)

3.
A vector field $\mathbf{F}(\mathbf{r})$ is equal to the curl of a vector potential $\mathbf{A}$ which is a continuous function of $\mathbf{r}$: $\mathbf{F}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$.

Prove that $\mathbf{F}_\perp$ is continuous across any surface, where "$\perp$" refers to the component of $\mathbf{F}$ which is perpendicular to the surface.

4.
A transverse electromagnetic wave in vacuum is normally incident on a semi-infinite slab of electrically insulating material which has $\epsilon = \epsilon_0$ but $\mu = \kappa_m \mu_0$. That is, the material has trivial dielectric but nontrivial magnetic properties. Consider the ferromagnetic limit $\kappa_m \to \infty$.

In that limit, what is the ratio of peak electric field $|\mathbf{E}''|_{\max}$ in the transmitted wave to the same quantity $|\mathbf{E}|_{\max}$ in the incident wave?

5.
Plane waves propagating in the $\pm z$ directions bounce between two semi-infinite nonconducting materials. The first material occupies the region $z < 0$, and the second occupies $z > L$, where L is a positive fixed distance. These (hypothetical) materials have no unusual magnetic properties ($\mu = \mu_0$), but they have *infinite* dielectric constant, $\epsilon/\epsilon_0 = \infty$.
(a.)
What components of $\mathbf{E}$ and $\mathbf{H}$ vanish, and where do they vanish? Justify each answer that you give.
(b.)
What angular frequencies for the light are possible?

6.
A nonrelativistic particle of mass m in one (x) dimension can be represented by a (complex) wavefunction

$$\Psi(x,t) \propto \exp\left(i(kx - \omega t)\right)$$

where

$$\hbar^2 k^2(x) = 2m(E - V(x)),$$

$E = \hbar\omega$ is the total and $V(x)$ is the potential energy, and $2\pi\hbar$ is Planck's constant. If $V(x)$ is piecewise flat, at the discontinuities in V the Schrödinger equation demands that *both* Ψ and $\partial\Psi/\partial x$ remain continuous.

Consider the reflection of a particle of initial kinetic energy T by a potential barrier of height $\Delta V < T$. Show that the ratio of reflected to incident amplitudes is given by the same formula as for the normal reflection of an electromagnetic wave at a dielectric interface with $\mu = \mu_0$ everywhere, provided that the refractive index in the quantum mechanical case is taken to be proportional to k.

7.
Fowles 3.1.

8.
Fowles 3.6.